Belfairs Academy

Mathematics Fundamentals Year 13 – Pure Mathematics

Knowledge	Skills
Algebraic Fractions	
• Simplification of rational expressions including factorising and cancelling. • Algebraic long division.	• To be able to simplify algebraic fractions where the denominators will be linear or quadratic, e.g. $\frac{1}{ax+b}, \frac{ax+b}{px^2+qx+r}, \frac{x^3+1}{x^2-1}$

Knowledge	Skills
Functions	
• To understand the definition of a function. • To know the domain and range of a function. • To calculate composition of functions. • To determine the inverse of a function. • To draw graphical representations of functions and their inverse. • To understand the modulus function	• One-one or many-one mapping from $\mathbb{R}$ (or a subset of $\mathbb{R}$) to $\mathbb{R}$. • The notation f : x and f(x) will be used. • To know that fg will mean 'do g first, then f '. • To understand that if f^{-1} exists, then $f^{-1}f(x) = ff^{-1}(x) = x$. • Students should be able to sketch the graphs of $y = \|ax + b\|$ and the graphs of $y = \|f(x)\|$ and $y = f(\|x\|)$, given the graph of y = f(x).

Knowledge	Skills
Exponential and Logarithm	
• The function e^x and its graph. • The function ln x and its graph • ln x as the inverse function of e^x.	• To apply knowledge to understand graphs of $y = e^{ax+b}$ • To find solutions of equations of the form $e^{ax+b} = p$ and $\ln(ax + b) = q$

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Knowledge	Skills
Numerical Methods	
• To find location of roots f(x) = 0 by considering changes of sign of f(x) in an interval of x in which f(x) is continuous. • To approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$	• To understand the terms continuous and discrete functions • To recognise that a change of sign indicates a root of an equation • To acknowledge that different iterative processes can give multiple solutions to equations.

Knowledge	Skills
Transforming graphs of functions	
• To understand and apply combinations of the transformations $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$. • This includes trigonometric graphs.	• Students should be able to sketch the graph of, for example, $y = 2f(3x)$, $y = f(-x) + 1$ given the graph of $y = f(x)$. • To be able to sketch the graph of, for example, $y = 3 + \sin 2x$, or $y = -\cos(x + \pi/4)$

Knowledge	Skills
Trigonometry	
• Knowledge of secant, cosecant and cotangent • Understanding of arcsin, arccos and arctan. • Know their relationships to sine, cosine and tangent. • To gain an understanding of their graphs and appropriate restricted domains.	• To understand that angles are measured in both degrees and radians • To draw trigonometric graphs using both degrees and radians.

Knowledge	Skills
Further trigonometric identities	
• Knowledge and use of $\sec^2\theta = 1 + \tan^2\theta$ • Knowledge and use of $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$.	• To include application to half angles. • To solve equations such as $a\cos\theta + b\sin\theta = c$ in a given interval

• Knowledge and use of double angle formulae • Use of formulae for sin $(A \pm B)$, cos $(A \pm B)$ and tan $(A \pm B)$ • To write expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm a)$ or $r\sin(\theta \pm a)$.	• To prove simple identities such as $\cos x \cos 2x + \sin x \sin 2x \equiv \cos x$.

Knowledge	**Skills**
Differentiation	
• To be able to differentiate e^x • To differentiation ln x • To differentiate sin(x), cos(x) and tan(x) • To apply the product rule • To apply the quotient rule • To apply the chain rule • To use dy/dx = 1/(dx/dy)	• To be able to differentiate multiple functions that include the sum and difference of trigonometric functions. • Differentiation of cosec(x), sec(x) and cot(x) • differentiation of functions generated from standard forms using products, quotients and composition, such as $2x^4 \sin x$, e^{3x}/x, $\cos x^2$ and $\tan^2 2x$. • To find dy/dx for x=sin(3y)

Knowledge	**Skills**
Partial fractions	
• To understand rational functions. • To understand partial fractions and split fractions where the denominators are not more complicated than repeated linear terms. • To apply skills to integration and differentiation, and series expansions.	• Recall how to add (algebraic) fractions with different denominators • To be able to work backwards and split an algebraic fraction into components called "Partial Fractions". • Partial fractions to include denominators such as $(ax + b)(cx + d)(ex + f)$ and $(ax + b)(cx + d)^2$.

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Knowledge	Skills
Co-ordinate geometry in the x-y plane	
• To understand the difference between Cartesian and parametric equations. • To write parametric equations of curves. • To convert between Cartesian and parametric forms.	• To be able to find the area under a curve given its parametric equations.

Knowledge	Skills
Binomial expansion	
• To be able to undertake a binomial expansion for any rational n.	• For $\|x\| < b/a$, students should be able to obtain the expansion of $(ax + b)n$ • To be able to expand rational functions by decomposition into partial fractions.

Knowledge	Skills
Further differentiation	
• To be able to differentiate simple functions defined implicitly or parametrically. • To apply differentiation to exponential growth or decay. • To be able to form and solve simple differential equations.	• To be able to find equations of tangents and normals to curves given parametrically or implicitly. • To gain a knowledge and apply the result of $d/dx(a^x) = a^x \ln a$ • To solve differential equations involving connected rates of change.

Knowledge	Skills
Vectors	
• To understand and apply vectors in two and three dimensions. • To know the magnitude of a vector. • To apply algebraic operations of vector addition, multiplication by scalar • To interpret geometrically algebraic operations of vectors. • To understand position vectors • To calculate the distance between two points	• To be able to find a unit vector in the direction of $\mathbf{a}$ • To be familiar and apply $\|\mathbf{a}\|$ • The distance d between two points (x_1 , y_1 , z_1) and (x_2 , y_2 , z_2) is given by $d^2 = (x_1 – x_2)^2 + (y_1 – y_2)^2 + (z_1 – z_2)^2$. • To include the forms $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + t(\mathbf{d} – \mathbf{c})$. • To calculate the intersection, or otherwise, of two lines. • Students should know that for $OA = \mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and

• To state the vector equation of lines. • To determine the scalar product and its application for calculating the angle between two lines.	$OB = \mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ and $\cos \angle AOB = \mathbf{a} \cdot \mathbf{b} / (\lvert\mathbf{a}\rvert \lvert\mathbf{b}\rvert)$ • To know that $\mathbf{a} \cdot \mathbf{b} = 0$, and $\mathbf{a}$ and $\mathbf{b}$ are nonzero vectors, then $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

Knowledge	**Skills**
Integration	
• To integrate e^x • To integrate 1/x • To integrate sin(x) and cos(x) • To integrate via substitution and by parts. • To apply simple cases of integration by partial fractions. • To analyse solutions of simple first order differential equations with separate variables. • To numerically integrate functions.	• To include integration of standard functions such as sin 3x, sec2 2x, tan(x), e^{5x}, 1/2x • To be able to use trigonometric identities to integrate, for example, $\sin^2 x$, $\tan^2 x$, $\cos^2 3x$. • To understand that integration by substitution and parts is the reverse of chain and product rules respectively. • To integrate ln x with respect to x • To apply integration by parts multiple times • To integrate rational expressions such as those arising from partial fractions. • To apply trapezium rule, recognising that increasing the number of trapezia improves accuracy.